The High-Frequency Resistance of Multiply-Stranded Insulated Wire.

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Introductory.

Conductors which have to carry high-frequency currents are often made up of a large number of separately insulated fine wires stranded together, with the object of compelling the current to distribute itself over the whole cross-section of the conductor. This may be done for two different reasons: firstly, to decrease the variation of inductance and resistance with change of frequency; and, secondly, to decrease the effective resistance of the conductor at high frequencies. To be effective, it is necessary that every wire should occupy in turn the same relative position in the conductor, so that the electromotive force induced in each wire by the magnetic flux should have the same average value over the whole length of the wire. If every separate wire has the same resistance, the same applied P.D. and the same induced E.M.F., both in amplitude and phase, they will all necessarily carry the same current, and the total current will therefore be uniformly distributed between all the wires.

The usual method of obtaining this similarity in the path of every strand is to make a conductor of three, four, or five wires twisted together, and then to twist three such conductors together, and so on until the resulting conductor contains the required number of wires. A large number of such multiple conductors are sometimes plaited or braided into a tubular conductor. Two of the individual wires of such a conductor may be in contact, except for the insulation, at a certain point, and then again further on at another point, one wire having followed an internal and the other an external path between the two points. From a knowledge of the current, frequency, and projected area of the loop formed by the two wires, measured normal to the magnetic flux, the E.M.F. induced in the loop can be calculated; thus, if the multiple conductor has a diameter of 1 cm. and is wound into a coil with one turn per centimetre, and if the current is 100 amperes at a frequency of 10^6, the E.M.F. induced in a loop of 5 sq. cm. is about 20 volts, giving 10 volts between the wires at each point of contact. In circuits containing spark-gaps the rate of change of the current may reach very high values and thus cause much greater potential differences between

the separate strands, which must therefore be efficiently insulated to prevent sparking between them.

A single copper wire 0·01 cm. diameter has a resistance, at a frequency of 500,000, differing very little from its resistance to continuous current. It must not be assumed, however, that the same is true of a conductor made up of a number of these fine insulated wires, however perfectly they may be stranded or plaited together. That the effective resistance of such conductors at high frequencies may be considerably greater than with continuous currents has been known for some time,* but little was known which would enable a designer of high-frequency apparatus to decide between a solid wire and different types of multiply-stranded wire. In this paper simple formulæ are established and Tables given by means of which the effective resistance of such conductors can be determined, and the size and number of strands chosen so as to give the best results under the specified conditions.

When a high-frequency current is passed through a single solid wire, the current is confined more and more to the outer part as the frequency is raised, leaving the inside free from both current and magnetic flux. When the wire is situated in the immediate neighbourhood of a number of other wires carrying high-frequency current it will be exposed to the magnetic field due to the latter, which will induce eddy-currents in it. Assuming the wires to be approximately parallel, these eddy-currents will pass up one side of the wire and down the other, adding to the energy dissipated in the wire, without affecting the total current passing through it. That the loss due to the eddy-currents is simply added to that due to the main current can be shown as follows. Let σ_1 be the uniform density of the main current, and σ_2 the density at any point of the superimposed eddy-current; the integral of the former over the cross-section of the wire is the main current, whilst that of the latter is zero. The loss of power per centimetre of the wire is $\int(\sigma_1+\sigma_2)^2\rho\, d\mathrm{S}$, where $d\mathrm{S}$ is an element of cross-section; this is equal to $\int\sigma_1^2\rho\, d\mathrm{S}+\int\sigma_2^2\rho\, d\mathrm{S}$, since $2\sigma_1\int\sigma_2\, d\mathrm{S}$ is zero. With alternating current this is true at every instant, so that the losses, as calculated from the root-mean-square values of the currents, may be added without any question as to the phase of the various currents.

The magnetomotive force due to the eddy-currents will tend to oppose the passage of the flux and to force it to pass on either side of the wire without going through it. For this reason, calculations based on the assumption that the distribution of the magnetic flux is unaltered by the eddy-currents are

* Meissner, 'Jahrbuch der Drahtlosen Telegraphie,' 1909, p. 57; Lindemann, 'Deut. Phys. Gesell.,' 1909, p. 682; 1910, p. 572; 'Jahrbuch der D. T.,' 1911, p. 561; Moller, 'Ann. der Physik,' vol. 36, p. 738 (1911); 'Jahrbuch der D. T.,' 1914, p. 32.

only applicable up to a certain point, beyond which the actual flux distribution must be taken into account. In the first part of this paper it is assumed that the flux distribution is unaltered by the eddy-currents which it produces, and the effective resistance of a multiply-stranded conductor both when straight and when coiled is calculated on this assumption. Formulæ are then established for the best space-factor to adopt when making such conductors and for the minimum resistance thereby obtained. The results are then tabulated for general use. In the second part of the paper, the problem is solved in a more general manner, taking into account the effect of the eddy-currents in modifying the magnetic field within the conductor; it is shown, however, that this precaution is unnecessary for the majority of the cases tabulated in Part I.

List of Symbols Employed.

All symbols referring to alternating magnitudes represent their maximum values unless otherwise stated.

α = space factor of conductor $= \dfrac{\text{cross-section of copper}}{\text{total cross-section}}$.

B = density of magnetic flux.

$d = 2r$ = diameter of each single strand, bare (cm.).

d_1 = diameter of each single strand, insulated (cm.).

D = diameter of multiple conductor (cm.).

f = frequency, cycles per second.

H = magnetic force at any point.

I = main current carried by multiple conductor (amperes).

$j = \sqrt{(-1)}$.

μ = permeability.

n = number of separate wires in multiple conductor.

$\omega = 2\pi f$.

Φ = magnetic flux.

ρ = specific resistance in ohms per centimetre cube.

R_o = resistance of multiple conductor to continuous current.

R_f = resistance of straight multiple conductor to alternating current.

R_c = resistance of coiled multiple conductor to alternating current.

R_{so} = resistance of solid conductor of same overall diameter to continuous current.

R_{sf} = resistance of solid conductor of same overall diameter to alternating current; conductor straight.

R_{sc} = resistance of solid conductor of same overall diameter to alternating current; conductor coiled.

S = turns per centimetre in coils.
σ = current density in amperes per square centimetre.
t = side of square individual strand (cm.).
τ = side of square multiple conductor (cm.).

PART I.

Consider a length of 1 cm. of a long solid wire of diameter d situated in an alternating magnetic field of strength H at right angles to the length of the wire. The E.M.F. induced in the rectangle d cm. wide and 1 cm. long (see fig. 1) is $\Delta E = \omega H d\, 10^{-8}$ volts, and the current density at the outer edges of the wire due to this induced E.M.F. is $\sigma = \Delta E/2\rho$ amperes per cm.². At any point in the wire at a distance y from the plane through its axis, parallel to H,

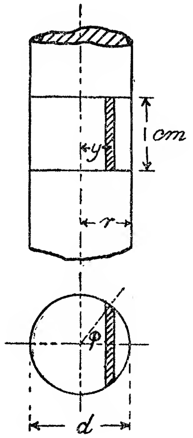

FIG 1

$$\sigma = \frac{\Delta E}{2\rho} \cdot \frac{y}{r} = \frac{\omega H y}{\rho} \cdot 10^{-8} \text{ amperes per cm.}^2$$

The loss per cubic centimetre due to this current is

$$\frac{\sigma^2 \rho}{2} = \frac{\omega^2 H^2 10^{-16}}{2\rho} \cdot y^2 \text{ watts,}$$

that in the strip of width dy, breadth $2r \sin\phi$, and length 1 cm. (see fig. 1) is

$$\frac{\omega^2 H^2 10^{-16}}{2\rho} \cdot y^2 \cdot 2r \sin\phi\, dy \text{ watts,}$$

and that in the whole wire per centimetre of length

$$\frac{H^2 \omega^2 \pi r^4 10^{-16}}{8\rho} \text{ watts.} \qquad (1)$$

Straight Stranded Conductor of Circular Section.

At any point within a straight conductor of circular section, at a distance x from the axis, the magnetic force due to its own current is $H = Ix/5R^2$. It is assumed that the individual strands are approximately parallel to the axis of the cable, in which case H is perpendicular to the axis of each strand as assumed in fig. 1. The average value of H^2 taken over the whole cross-section of the cable is

$$\frac{1}{\pi R^2} \int_0^R H^2 \cdot 2\pi x \cdot dx = \frac{I^2}{50R^2}. \qquad (2)$$

Substituting this in equation (1), we find for the average loss in a entimetre length of single wire,

$$\frac{I^2\omega^2\pi r^4}{400\rho R^2} \cdot 10^{-16} \text{ watts.}$$

This is due only to the self-induced eddy-currents; the loss due to the main current I/n in 1 cm. of single wire is

$$\frac{I^2}{2n^2} \cdot \frac{\rho}{\pi r^2} \text{ watts.}$$

Hence the total loss in 1 cm. of single wire is

$$\frac{I^2}{2n^2}\left(\frac{\rho}{\pi r^2} + \frac{\omega^2\pi r^4 n^2 10^{-16}}{200\rho R^2}\right).$$

In 1 cm. of the whole cable the loss is n times as great.

Hence
$$R_f = R_o\left(1 + \frac{\omega^2\pi^2 r^6 n^2 10^{-16}}{200\rho^2 R^2}\right), \tag{3}$$

which, for copper ($\rho = 1{\cdot}7 \cdot 10^{-6}$) may be written

$$\begin{aligned} R_f &= R_o(1 + 4{\cdot}2 \cdot 10^{-6} \cdot n^2 f^2 d^6/D^2), \\ &= R_o(1 + 4{\cdot}2 \cdot 10^{-6} \cdot n f^2 d^4 \alpha), \end{aligned} \tag{3a}$$

where $\alpha = nd^2/D^2$ is the space factor.

Closely-Wound Long Single-Layer Coil.

It is assumed for simplicity that the conductor has a square cross-section, and that adjacent turns are in actual contact, as shown in fig. 2. Let x be

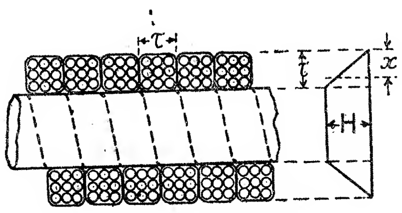

FIG 2

the distance from the outer surface of any point within the conductor, then at this point

$$H = \frac{4\pi IS}{10} \cdot \frac{x}{\tau}$$

and the average value of H^2 over the whole cross-section of the square conductor is

$$\frac{16\pi^2 I^2 S^2}{100} \cdot \frac{1}{3}, \tag{4}$$

which, substituted in equation (1), gives for the average loss in a centimetre length of single wire

$$\frac{\pi^3 I^2 S^2 \omega^2 r^4 10^{-16}}{150\rho} \text{ watts.}$$

For 1 cm. of the whole conductor it will be n times as great. This is the loss due to the self-induced eddy-currents only, the total loss in 1 cm. length of the stranded conductor is

$$\frac{I^2}{2} \cdot \frac{\rho}{n\pi r^2} + \frac{I^2 S^2 \omega^2 r^4 n \pi^3 10^{-16}}{150\,\rho} \text{ watts.}$$

Hence

$$R_c = R_o\left(1 + \frac{4\pi^4 r^6 \omega^2 S^2 n^2 10^{-18}}{3\rho^2}\right). \tag{5}$$

For copper, this may be written

$$R_c = R_o(1+28 \,.\, 10^{-6} n^2 f^2 S^2 d^6)$$
$$= R_o(1+35{\cdot}6 \,.\, 10^{-6} n f^2 d^4 \alpha), \tag{5a}$$

since $\alpha\tau^2 = n\pi d^2/4$ and, if the adjacent turns are in contact, $S\tau = 1$.

If not closely wound, *i.e.*, if $S\tau < 1$, $S^2 d^2 = (S\tau)^2/n \,.\, d^2/d_1^2$, since, assuming the small wires to be arranged as shown in fig. 2, $\tau^2 = nd_1^2$. If the space between adjacent turns is small, the formula for H will still be approximately correct, and

$$R_c = R_o[1+35{\cdot}6 \,.\, 10^{-6} \,.\, nf^2 d^4 \alpha (S\tau)^2]. \tag{5b}$$

As an example of the use of this formula, it can be applied to an experimental result obtained by Lindemann (*loc. cit.*). He found that 14 turns of stranded conductor wound on a glass cylinder 24 cm. diameter, had a resistance at a frequency of 900,000 cycles per second, 15 times its continuous current value. The conductor consisted of 180 enamelled wires 0·012 cm. diameter, arranged $3 \times 3 \times 4 \times 5$, *i.e.*, five single wires were twisted together, then four of these conductors, and so on. For a single wire 0·012 cm. diameter at a frequency of 900,000, $d\sqrt{f} = 11{\cdot}4$, from which it can be shown that if wound singly into a solenoid, the increase in resistance due to skin effect would be about 10 per cent. Applying the formula just found for a stranded wire solenoid, viz.:

$$R_c = R_o\{1+35{\cdot}6 \,.\, 10^{-6} \,.\, nf^2 d^4 \alpha (S\tau)^2\},$$

assuming $\alpha = 0{\cdot}5$ and $S\tau = 0{\cdot}9$, since, although the turns are in contact, the conductor is round and not square, we have

$$R_c = R_o(1+43{\cdot}1).$$

Now the above expression applies only to a very long solenoid, whereas the coil tested by Lindemann was a narrow one of large diameter, for which it can be readily shown that the magnetic field strength at the inner side of

the wire is little more than half that in a long solenoid. The eddy-current losses would therefore be a little more than a quarter of those calculated for a long solenoid. This agrees approximately with the value $R_c = 15\ R_o$ found experimentally.

Now a solid wire with the same cross-section of copper has a diameter of 0·16 cm. At the same frequency $d\sqrt{f}$ would be 153, and R_c/R_o for a long solenoid would be about 13 or 14. Hence, in this case, the stranded conductor is much inferior to the solid wire with the same cross-section of copper; in the same space, however, a solid wire of greater cross-section could be wound, thus further reducing the resistance, whilst the cost would be but a fraction of that of the stranded wire.

It will be seen from equations (3*a*) and (5*a*) that, for given values of d and α, the ratio $(R_f - R_o)/R_o$ of the increase in resistance to the continuous current resistance is proportional to n, the number of wires in the conductor; hence the difficulty increases with the current for which the conductor is to be designed. If the total cross-section, including insulation and air space, remains unchanged, and the space factor is maintained constant, n will vary inversely as d^2, and the percentage increase of resistance will be proportional to d^2 or inversely proportional to n. The diameter of the individual strands, however, can hardly be reduced below about 0·005 cm., on account of manufacturing difficulties.

If the overall diameter of the cable and the diameter of the wire to be employed are fixed, it is important to note that it is not necessarily beneficial to crowd as many wires as possible into the available space. The best value of n or of the space factor α can be calculated in each case as follows. It has been shown that

$$R_f = R_o(1 + knf^2d^4\alpha),$$

where $k = 4{\cdot}2 \,.\, 10^{-6}$ for a straight conductor of copper and $35{\cdot}6 \,.\, 10^{-6}$ for a very closely wound long solenoid with a conductor of square section. Now, if the resistance to continuous current of a solid wire of the same overall diameter is R_{so},

$$\alpha = \frac{R_{so}}{R_o}, \qquad \frac{n}{\alpha} = D^2/d^2,$$

and

$$R_f/R_{so} = \frac{1}{\alpha} + knf^2d^4,$$

$$= \frac{D^2}{nd^2} + knf^2d^4. \tag{6}$$

For this to be a minimum with given values of D and d, it is necessary that

$$n = \frac{1}{\sqrt{k}} \cdot \frac{D}{fd^3}. \tag{7}$$

If n is made greater than this, the resistance to high-frequency currents will not be decreased but increased. As an example, a conductor 0·5 inch diameter, made up of wires 0·01 cm. diameter, for a frequency of 500,000, should have $n = 1240$, which corresponds to a space factor of only 0·077.

The best value of the space-factor in any given case is given by the equation,

$$\alpha = \frac{nd^2}{D^2} = \frac{1}{\sqrt{k}} \cdot \frac{1}{fdD}. \tag{8}$$

If the space-factor has this optimum value, then

$$R_f/R_o = 1 + kn\alpha f^2 d^4 = 1 + 1 = 2, \tag{9}$$

and
$$R_f/R_{so} = R_f/R_o \times R_o/R_{so} = \frac{2}{\alpha} = 2\sqrt{k} \,.\, fdD. \tag{10}$$

On substituting in equations (8) and (10) the values found above for k, it is seen that, for a straight conductor of copper, the optimum space factor is

$$\frac{487}{fdD}, \tag{11}$$

and
$$R_f/R_{so} = \frac{2fdD}{487}, \tag{12}$$

whilst for a single-layer solenoid wound with a conductor of square section, with $S_T = 0{\cdot}71$, $k = 17{\cdot}8 \,.\, 10^{-6}$, the optimum space-factor

$$= \frac{238}{fdD}, \tag{13}$$

and
$$R_c/R_{so} = \frac{2fdD}{238}. \tag{14}$$

[*Note.*—If it is assumed that round conductor is equivalent to square of the same cross-section, then $S_T = 0{\cdot}88$ SD, and equations (13) and (14) are applicable to round conductors with SD = 0·8].

These values of R_f/R_{so} are only true if the conductor is made with the optimum space factor, and this depends on the frequency, and on whether the wire is straight or coiled, and, in the latter case, on the details of the winding. The values of the ratios

$$A = \frac{\text{A.C. resistance of stranded cable}}{\text{C.C. resistance of stranded cable}} = \frac{R_f}{R_o} \quad \text{or} \quad \frac{R_c}{R_o},$$

$$B = \frac{\text{A.C. resistance of stranded cable}}{\text{C.C. resistance of solid conductor}} = \frac{R_f}{R_{so}} \quad \text{or} \quad \frac{R_c}{R_{so}},$$

$$C = \frac{\text{A.C. resistance of solid conductor}}{\text{C.C. resistance of solid conductor}} = \frac{R_{sf}}{R_{so}} \quad \text{or} \quad \frac{R_{sc}}{R_{so}},$$

have been calculated for a number of typical cases, and the results are given

in the Tables. The solid conductor is assumed to have the same overall diameter as the stranded cable, and therefore capable of replacing it in any coil or apparatus. Four values of the overall diameter D have been taken, viz., 0·1, 0·2, 0·5, and 1 cm., and three alternative sizes of wire, viz., 0·005, 0·01, and 0·02 cm. The wave-lengths chosen are 300, 600, 1200, 3000, and 6000 metres. Below the heavy stepped line the space factors have been assumed to have the calculated optimum value; above the stepped line this calculated value was considered impossibly high, and the resistance has therefore been calculated for three alternative values of α, viz., 0·1, 0·25, and 0·5. In such cases it is best to make α as large as possible. The coils are assumed to be long solenoids, with a single layer of square conductor, having a ratio of side to pitch of 0·71, which is approximately equivalent to round conductor, with a diameter equal to 0·8 of the pitch. With a more open winding or with shorter coils, the results would lie between these and those obtained for the same conductor when straight.

A comparison of the columns headed B and C in the Tables shows in each case the advantage, if any, to be gained by using insulated stranded conductor. The ratio of B to C is the ratio of the high-frequency resistance of the strand to that of the solid wire, which could be used in its place. If $d = 0·02$ cm., B is larger than C for wave-lengths below 1200 metres, but smaller than C for longer wave-lengths, that is to say that, even if made with the ideal space factor, the stranded conductor is inferior to the solid wire at the shorter wave-lengths. If $d = 0·01$ cm., the stranded conductor, if correctly made for the given conditions, is on a par with the solid conductor at short wave-lengths, and may be considerably better at longer wave-lengths. With the finest wire, viz., $d = 0·005$ cm., the stranded conductor may have a resistance much lower than that of the solid wire.

It will be noticed that for a solenoid wound with stranded conductor of 1·0 cm. diameter, made up of wires of 0·02 cm. diameter, the lowest resistance at a wave-length of 300 metres is obtained by making the space-factor 0·012, and that, even if made with this optimum space-factor, the high-frequency resistance is 168 times as great as the continuous-current resistance of a solid conductor of the same size, whereas the high-frequency resistance of the latter would be only 100 times its continuous-current value. The solid conductor could be replaced by a thin tube, since the depth of penetration into copper at a frequency of 10^6 is only about 0·03 cm. Hence it appears that it is only by using wires as small as 0·005 cm., and by stranding them together with a certain space-factor, that conductors for a frequency of a million can be made to have a lower effective resistance than solid wires or tubes occupying the same space.

Table I

Straight Conductor. $d = 0{\cdot}005$ cm.

D.	$f = 10^6$. $\lambda = 300$ m.				$f = 5 \times 10^5$. $\lambda = 600$ m.				$f = 2{\cdot}5 \times 10^5$. $\lambda = 1200$ m.				$f = 10^5$. $\lambda = 3000$ m.				$f = 5 \times 10^4$. $\lambda = 6000$ m.			
	α.	A.	B.	C.	α.	A.	B.	C.	α	A.	B.	C.	α.	A.	B.	C.	α.	A.	B.	C.
cm. 0·1	0·1	1·01	10·1	4·05	0·1	1·00	10·0	2·94	0·1	1	10	2·15	0·1	1	10	1·45	0·1	1	10	1·16
	0·25	1·06	4·3	4·05	0·25	1·01	4·06	2·94	0·25	1	4	2·15	0·25	1	4	1·45	0·25	1	4	1·16
	0·5	1·26	2·5	4·05	0·5	1·06	2·13	2·94	0·5	1	2	2·15	0·5	1	2	1·45	0·5	1	2	1·16
0·2	0·48	2	4·1	7·85	0·1	1·01	10·1	5·62	0·1	1	10	4·05	0·1	1	10	2·65	0·1	1	10	1·95
					0·25	1·06	4·26	5·62	0·25	1·01	4·06	4·05	0·25	1	4	2·65	0·25	1	4	1·95
					0·5	1·26	2·52	5·62	0·5	1·06	2·13	4·05	0·5	1	2	2·65	0·5	1	2	1·95
0·5	0·2	2	10·2	19·2	0·4	2	5·1	13·6	0·1	1·02	10·2	9·75	0·1	1	10	6·25	0·1	1	10	4·5
									0·25	1·1	4·4	9·75	0·25	1	4	6·25	0·25	1	4	4·5
									0·5	1·4	2·8	9·75	0·5	1·05	2·1	6·25	0·5	1	2	4·5
1·0	0·1	2	20·5	38·2	0·2	2	10·2	27·1	0·4	2	5·1	19·2	0·1	1	10	12·2	0·1	1	10	8·75
													0·25	1·06	4·25	12·2	0·25	1·02	4·1	8·75
													0·5	1·25	2·5	12·2	0·5	1·05	2·1	8·75

Table II.

Straight Conductor. $d = 0{\cdot}01$ cm.

D.	$f = 10^6$. $\lambda = 300$ m.				$f = 5 \times 10^5$. $\lambda = 600$ m.				$f = 2{\cdot}5 \times 10^5$. $\lambda = 1200$ m.				$f = 10^5$. $\lambda = 3000$ m.				$f = 5 \times 10^4$. $\lambda = 6000$ m.			
	a.	A.	B.	C.	*a.*	A.	B.	C.	*a.*	A.	B.	C.	*a.*	A.	B.	C.	*a.*	A.	B.	C.
cm. 0·1	0·49	2	4·1	4·05	0·1	1·01	10·1	2·94	0·1	1	10	2·15	0·1	1	10	1·45	0·1	1	10	1·16
					0·25	1·06	4·26	2·94	0·25	1·01	4·06	2·15	0·25	1	4	1·45	0·25	1	4	1·16
					0·5	1·26	2·52	2·94	0·5	1·06	2·13	2·15	0·5	1	2	1·45	0·5	1	2	1·16
0·2	0·24	2	8·2	7·85	0·49	2	4·1	5·62	0·1	1·01	10·1	4·05	0·1	1	10	2·65	0·1	1	10	1·95
									0·25	1·06	4·26	4·05	0·25	1	4	2·65	0·25	1	4	1·95
									0·5	1·26	2·52	4·05	0·5	1·05	2·1	2·65	0·5	1	2	1·95
0·5	0·1	2	20·5	19·2	0·2	2	10·2	13·6	0·4	2	5·12	9·75	0·1	1	10	6·25	0·1	1	10	4·5
													0·25	1·06	4·26	6·25	0·25	1·02	4·1	4·5
													0·5	1·26	2·52	6·25	0·5	1·05	2·1	4·5
1·0	0·05	2	41	38·2	0·1	2	20·5	27·1	0·2	2	10·2	19·2	0·49	2	4·1	12·2	0·1	1	10	8·75
																	0·25	1·06	4·26	8·75
																	0·5	1·26	2·52	8·75

Table III.

Straight Conductor. $d = 0\cdot02$ cm.

D.	$f = 10^6$. $\lambda = 300$ m.				$f = 5 \times 10^5$. $\lambda = 600$ m.				$f = 2\cdot5 \times 10^5$. $\lambda = 1200$ m.				$f = 10^5$. $\lambda = 3000$ m.				$f = 5 \times 10^4$. $\lambda = 6000$ m.			
	α.	A.	B.	C.	α.	A.	B.	C.	α.	A.	B.	C.	α.	A.	B.	C.	α.	A.	B.	C.
cm. 0·1	0·24	2	8·2	4·05	0·48	2	4·1	2·94	0·1 0·25 0·5	1·01 1·06 1·26	10·1 4·26 2·52	2·15 2·15 2·15	0·1 0·25 0·5	1 1 1·05	10 4 2·1	1·45 1·45 1·45	0·1 0·25 0·5	1 1 1	10 4 2	1·16 1·16 1·16
0·2	0·12	2	16·4	7·85	0·24	2	8·2	5·62	0·48	2	4·1	4·05	0·1 0·25 0·5	1·01 1·05 1·15	10·1 4·2 2·3	2·65 2·65 2·65	0·1 0·25 0·5	1 1 1·05	10 4 2·1	1·95 1·95 1·95
0·5	0·05	2	41	19·2	0·1	2	20·5	13·6	0·2	2	10·2	9·75	0·4	2	4·1	6·25	0·1 0·25 0·5	1 1·06 1·26	10 4·26 2·52	4·5 4·5 4·5
1·0	0·025	2	82	38·2	0·05	2	41	27·1	0·1	2	20·5	19·2	0·24	2	8·2	12·2	0·48	2	4·1	8·75

Table IV.

Single Layer Long Solenoid. $d = 0{\cdot}005$ cm.

D.	$f = 10^6$. $\lambda = 300$ m.				$f = 5\times10^5$. $\lambda = 600$ m.				$f = 2{\cdot}5\times10^5$. $\lambda = 1200$ m.				$f = 10^5$. $\lambda = 3000$ m.				$f = 5\times10^4$. $\lambda = 6000$ m.			
	a.	A.	B.	C.	a.	A.	B.	C.	a.	A.	B.	C.	a.	A.	B.	C.	a.	A.	B.	C.
cm. 0·1	0·49	2	4·2	9·6	0·1	1·01	10·1	6·8	0·1	1	10	4·7	0·1	1	10	3	0·1	1	10	2·2
					0·25	1·07	4·3	6·8	0·25	1·02	4·1	4·7	0·25	1	4	3	0·25	1	4	2·2
					0·5	1·27	2·5	6·8	0·5	1·05	2·1	4·7	0·5	1	2	3	0·5	1	2	2·2
0·2	0·24	2	8·4	20	0·49	2	4·2	14	0·1	1·01	10·1	9·6	0·1	1	10	6	0·1	1	10	4·2
									0·25	1·07	4·3	9·6	0·25	1	4	6	0·25	1	4	4·2
									0·5	1·27	2·5	9·6	0·5	1·05	2·1	6	0·5	1	2	4·2
0·5	0·1	2	21	50	0·2	2	10·5	35	0·4	2	5·2	25	0·1	1·01	10·1	16	0·1	1	10	11
													0·25	1·07	4·3	16	0·25	1·02	4·1	11
													0·5	1·27	2·5	16	0·5	1·05	2·1	11
1·0	0·05	2	42	100	0·1	2	21	71	0·2	2	10·5	50	0·49	2	4·2	32	0·1	1·01	10·1	22
																	0·25	1·07	4·3	22
																	0·5	1·27	2·5	22

Table V.

Single Layer Long Solenoid. $d = 0{\cdot}01$ cm.

D.	$f = 10^6$. $\lambda = 300$ m.				$f = 5 \times 10^5$. $\lambda = 600$ m.				$f = 2{\cdot}5 \times 10^5$. $\lambda = 1200$ m.				$f = 10^5$. $\lambda = 3000$ m.				$f = 5 \times 10^4$. $\lambda = 6000$ m.			
	a.	A.	B.	C.	a.	A.	B.	C.	a.	A.	B.	C.	a.	A.	B.	C.	a.	A.	B.	C.
cm. 0·1	0·24	2	8·4	9·6	0·49	2	4·2	6·8	0·1 0·25 0·5	1·01 1·07 1·27	10·1 4·3 2·5	4·7 4·7 4·7	0·1 0·25 0·5	1 1 1·05	10 4 2·1	3 3 3	0·1 0·25 0·5	1 1 1	10 4 2	2·2 2·2 2·2
0·2	0·12	2	16·8	20	0·24	2	8·4	14	0·49	2	4·2	9·6	0·1 0·25 0·5	1·01 1·05 1·18	10·1 4·2 2·3	6 6 6	0·1 0·25 0·5	1 1 1·05	10 4 2·1	4·2 4·2 4·2
0·5	0·05	2	42	50	0·1	2	21	35	0·2	2	10·5	25	0·49	2	4·2	16	0·1 0·25 0·5	1·01 1·07 1·27	10·1 4·3 2·5	11 11 11
1·0	0·025	2	84	100	0·05	2	42	71	0·1	2	21	50	0·25	2	8·4	32	0·49	2	4·2	22

Table VI.

Single Layer Long Solenoid. $d = 0{\cdot}02$ cm.

D.	$f = 10^6$. $\lambda = 300$ m.				$f = 5 \times 10^5$. $\lambda = 600$ m.				$f = 2{\cdot}5 \times 10^5$. $\lambda = 1200$ m.				$f = 10^5$. $\lambda = 3000$ m.				$f = 5 \times 10^4$. $\lambda = 6000$ m.			
	α.	A.	B.	C.	α.	A.	B.	C.	α.	A.	B.	C.	α.	A.	B.	C.	α.	A.	B.	C.
cm. 0·1	0·12	2	16·8	9·6	0·24	2	8·4	6·8	0·49	2	4·2	4·7	0·1 0·25 0·5	1·01 1·05 1·18	10·1 4·2 2·3	3 3 3	0·1 0·25 0·5	1 1 1·05	10 4 2·1	2·2 2·2 2·2
0·2	0·06	2	33·6	20	0·12	2	16·8	14	0·24	2	8·4	9·6	0·49	2	3·4	6	0·1 0·25 0·5	1·01 1·05 1·18	10·1 4·2 2·3	4·2 4·2 4·2
0·5	0·025	2	84	50	0·05	2	42	35	0·1	2	21	25	0·24	2	8·4	16	0·49	2	4·2	11
1·0	0·012	2	168	100	0·025	2	84	71	0·05	2	42	50	0·12	2	16·8	32	0·24	2	8·4	22

The following Tables illustrate the effect of varying—

(a) The space-factor α, by varying n, whilst keeping D and d constant;

(b) The number of wires n, and therefore also D, whilst keeping d and α constant; and

(c) The frequency f only.

Table VII.—Straight Conductor; $D = 0{\cdot}5$ cm.; $d = 0{\cdot}01$ cm.; $f = 5 \times 10^5$.

α.	$A = R_f/R_o$.	$B = R_f/R_{so}$.	$C = R_{sf}/R_{so}$.
0·05	1·06	21·3	13·6
0·1	1·26	12·6	13·6
0·2	2·05	10·2	13·6
0·3	3·36	11·2	13·6
0·5	7·55	15·1	13·6

Table VIII.—Straight Conductor; $d = 0{\cdot}01$ cm.; $\alpha = 0{\cdot}3$; $f = 5 \times 10^5$.

n.	D.	$A = R_f/R_o$.	$B = R_f/R_{so}$.	$C = R_{sf}/R_{so}$.
	cm.			
30	0·1	1·10	3·65	2·94
120	0·2	1·38	4·59	5·62
300	0·32	1·95	6·48	8·74
750	0·5	3·36	11·2	13·65
3000	1·0	10·5	34·8	27·15

Table IX.—Straight Conductor; $D = 0{\cdot}5$ cm.; $d = 0{\cdot}01$ cm.; $\alpha = 0{\cdot}4$; $n = 1000$.

f.	$A = R_f/R_o$.	$B = R_f/R_{so}$.	$C = R_{sf}/R_{so}$.
10^6	17·8	44·5	19·2
$5 \cdot 10^5$	5·2	13	13·6
$2{\cdot}5 \cdot 10^5$	2·05	5·1	9·75
10^5	1·17	2·9	6·25
$5 \cdot 10^4$	1·04	2·6	4·5

From Table VII it is seen that for the conditions there assumed the best space-factor is 0·2, but the stranded is slightly superior to the solid conductor over a wide range of space-factors. Table VIII shows the stranded conductor to be inferior for both extreme values, viz., 30 and 3000 wires, but slightly better than the solid for intermediate values. Table IX shows how the superiority of the stranded conductor at low frequencies disappears as the frequency is raised.

As an example of a type of stranded conductor largely employed, and of

the space-factor obtained, the following data are of interest: $3 \times 3 \times 3 = 27$ wires each, No. 36 S.W.G. ($d = 0{\cdot}019$), each wire single silk covered, and the whole double silk covered; $D = 0{\cdot}16$ cm.; $\alpha = 0{\cdot}39$. Tables III and VI show that this wire has a higher resistance than the solid wire which could replace it over the whole range of radio-telegraphic frequencies, with the possible exception of closely-wound solenoids at very long wave-lengths.

Strictly speaking, D should not include the insulation of the multiple conductor; this would give a somewhat larger value of α, but would not modify the conclusions.

Part II.

In the first part it was assumed that the distribution of the magnetic flux throughout the stranded conductor is not appreciably affected by the eddy-currents in the individual wires. It is now to be seen whether this assumption is justified.

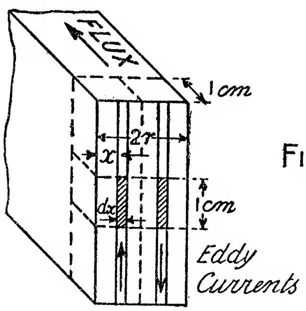

FIG 3

Fig. 3 shows a sectional view of a thin metal sheet, along which an alternating magnetic flux is maintained by some externally applied magneto-motive force. The thickness of the sheet is $2r$, and a piece of the metal 1 cm. long and 1 cm. deep is indicated. If $\delta\Phi$ is the magnetic flux at any moment through the rectangular strip of width δx and length 1 cm., then the flux density B is $\delta\Phi/\delta x$, and the E.M.F. around the strip is $-\frac{\partial}{\partial t}(\delta\Phi)10^{-8}$ volts. If the total upward electric force at a point in the metal is E,

$$\frac{\partial E}{\partial x} = \frac{\partial B}{\partial t} \cdot 10^{-8}.$$

If the current density is σ, then $\sigma = \frac{E}{\rho}$ and $\frac{\partial \sigma}{\partial x} = \frac{1}{\rho} \cdot \frac{\partial B}{\partial t} \cdot 10^{-8}$.

It is obvious from considerations of symmetry that the current distribution is the same on both sides of the centre line, along which σ is zero.

Let H_1 be the applied magnetic force, assumed to be uniform over the whole cross-section, and let H_2 be the magnetic force due to the currents in the plate; then as x increases to $x+\delta x$, H_2 will increase by an amount $4\pi\sigma\delta x/10$, and

$$\frac{\partial H_2}{\partial x} = \frac{4\pi}{10}\sigma.$$

Since $B = \mu(H_1+H_2)$, $\frac{\partial B}{\partial x} = \mu \frac{\partial H^2}{\partial x} = \frac{4\pi}{10}\mu\sigma.$*

From the two formulæ

$$\frac{\partial \sigma}{\partial x} = \frac{1}{10^8\rho} \cdot \frac{\partial B}{\partial t} \quad \text{and} \quad \frac{\partial B}{\partial x} = \frac{4\pi}{10}\mu\sigma,$$

it follows that

$$\frac{\partial^2 B}{\partial x^2} = a_1{}^2 \frac{\partial B}{\partial t} \quad \text{and} \quad \frac{\partial^2 \sigma}{\partial x^2} = a_1{}^2 \frac{\partial \sigma}{\partial t},$$

where

$$a_1{}^2 = \frac{4\pi\mu}{10^9\rho}.$$

Assuming a sine law or simple harmonic variation with respect to the time, and adopting symbolic notation

$$\frac{\partial B}{\partial t} = j\omega B \quad \text{and} \quad \frac{\partial \sigma}{\partial t} = j\omega\sigma,$$

and the above equations may be written

$$\frac{\partial^2 B}{\partial x^2} = a^2 B \quad \text{and} \quad \frac{\partial^2 \sigma}{\partial x^2} = a^2\sigma, \tag{15}$$

where

$$a^2 = j\omega \frac{4\pi\mu}{10^9\rho}.$$

B and σ are now vector or complex quantities and to find the actual value at any moment, it is necessary to find the vertical component of the vector, or the imaginary part of the complex quantity.

Putting $a = \beta + j\alpha'$, we find

$$\alpha' = \beta = 2\pi\sqrt{(f\mu/10^9\rho)}.$$

Equations (15) are identical with those found for the distribution of B and σ in the conductor when two flat parallel strips serve as the leads in an alternate current transmission. By solving equation (15) it can easily be shown that, if the frequency is very high, or the plate so thick that the penetration is small, $\sigma = \sigma_1\epsilon^{-\beta x}$, where σ_1 is the current density at the surface from which the depth x is measured; similarly $B = B_1\epsilon^{-\beta x}$, where $B_1 = \mu H_1$, since at the surface H_2 is zero and the resultant magnetic force H is equal to the applied force H_1.

All the energy dissipated within the plate is transmitted normally into it from the two surfaces and can be calculated by finding the energy entering each square centimetre of the surface, without any consideration of the distribution of current within the plate. It is interesting to note that the

* In Part II, up to the adoption of symbolic notation in equation (15), the symbols H, B, and σ, represent instantaneous values.

currents and losses within the plate would not be affected in any way by splitting the plate into two of half the thickness, and inserting between the two halves a very thin sheet of infinitely good conducting material. Similarly thin layers of infinitely good conducting material could be embedded at distances of 1 cm. as shown by the dotted lines in fig. 4. Now these fictitious

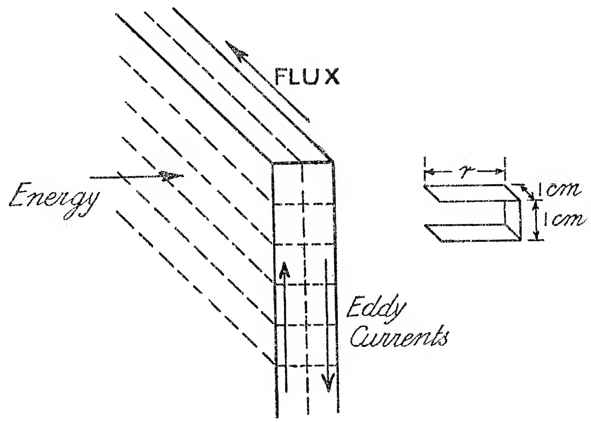

FIG 4

partitions form absolutely impenetrable reflectors of electromagnetic energy, and all the energy dissipated in each compartment must therefore have been transmitted into it from the surface. This transmission of energy into and through the metal plate is governed by the same laws as the transmission of energy through air or other dielectric in the case of a telephone line or alternate current power transmission system, and may be calculated directly by the usual transmission line formulæ. The "transmission line" into each compartment of the metal sheet is of length r, *i.e.*, half the thickness of the plate; it is short-circuited at the far end, viz., at the middle of the plate; its conductors, being of infinitely good conducting material, have no resistance, and its "dielectric," or, rather, the material through which the energy is transmitted, is copper or iron or other metal of which the plate may be made. The actual eddy-currents in the plate are the leakage currents of the fictitious transmission line. Considering a width of 1 cm. in the direction of the flux, as shown in the small sketch in fig. 4, the inductance L per centimetre of length of line, *i.e.* of penetration, is $4\pi\mu/10^9$ henries, and the leakage G is $1/\rho$.

For a line of length r, short-circuited at the receiving end,

$$i_s = i_r \cosh ar \qquad \text{and} \qquad V_s = i_r\sqrt{(Z/Y)}\sinh ar,$$

where the suffixes s and r refer to the sending and receiving ends respectively,

and Z and Y are the impedance and admittance per unit length of the line. The sending end impedance of the line

$$= \frac{V_s}{i_s} = \sqrt{\frac{Z}{Y}} \tanh ar.$$

If P is the power supplied to the line

$$P = \frac{i_s^2}{2} \times (\text{real part of the sending end impedance}).$$

In the present case $\sqrt{(Z/Y)} = \sqrt{(j\omega L/G)} = \sqrt{(4\pi\omega\rho\mu 10^{-9})}\,\underline{|45^\circ}$,

$$= \sqrt{2\beta\rho}\,\underline{|45^\circ},$$

and $$\tanh ar = \sqrt{\frac{\cosh 2\beta r - \cos 2\beta r}{\cosh 2\beta r + \cos 2\beta r}}\,\underline{|\phi - \theta},$$

where $\tan\phi = \tan\beta r/\tanh\beta r$ and $\tan\theta = \tan\beta r \,.\, \tanh\beta r$.

Hence the sending end impedance of the line

$$= \sqrt{2\beta\rho}\sqrt{\frac{\cosh 2\beta r - \cos 2\beta r}{\cosh 2\beta r + \cos 2\beta r}}\,\underline{|\phi - \theta + 45^\circ},$$

and since

$$\cos(\phi - \theta + 45^\circ) = \frac{1}{\sqrt{2}} \cdot \frac{\sinh 2\beta r - \sin 2\beta r}{\sqrt{(\sinh^2 2\beta r + \sin^2 2\beta r)}},$$

$$P = \frac{i_s^2}{2} \,.\, \beta\rho\, F(\beta t),$$

where $t = 2r$ = thickness of the sheet, and

$$F(\beta t) = \frac{\cosh\beta t - \cos\beta t}{\cosh\beta t + \cos\beta t} \cdot \frac{\sinh\beta t - \sin\beta t}{\sqrt{(\sinh^2\beta t + \sin^2\beta t)}}.$$

Introducing the field strength H_1 at the outer face, instead of the fictitious line current i_s, since

$$H_1 = 4\pi i_s/10,$$

$$P = \frac{H_1^2}{2} \cdot \left(\frac{10}{4\pi}\right)^2 \beta\rho\, F(\beta t) \text{ watts},$$

where H_1 is the maximum value of the applied magnetic force.

This is the power transmitted into and dissipated within a column of the material of 1 sq. cm. cross-section, and of length r or $t/2$. This is equivalent to

$$P = H_1^2 \left(\frac{10}{4\pi}\right)^2 \cdot \frac{\beta\rho}{t} \cdot F(\beta t) \text{ watts per cubic centimetre.} \qquad (16)$$

If βt is so small that all powers above the cube can be neglected, $F(\beta t) = \beta^3 t^3/6$, and

$$P = \frac{H_1^2 \omega^2 \mu^2 t^2}{\rho \,.\, 24 \,.\, 10^{16}} \text{ watts per cubic centimetre.}$$

This agrees exactly with the value obtained directly on the assumption that the magnetic field is not appreciably affected by the eddy currents.

This approximate value of F (βt) can only be used if βt is small; for $\beta t = 1$ it is 4 per cent. too high. F (βt) has been calculated, and the results plotted in fig. 5, together with the values of $\beta^3 t^3/6$.

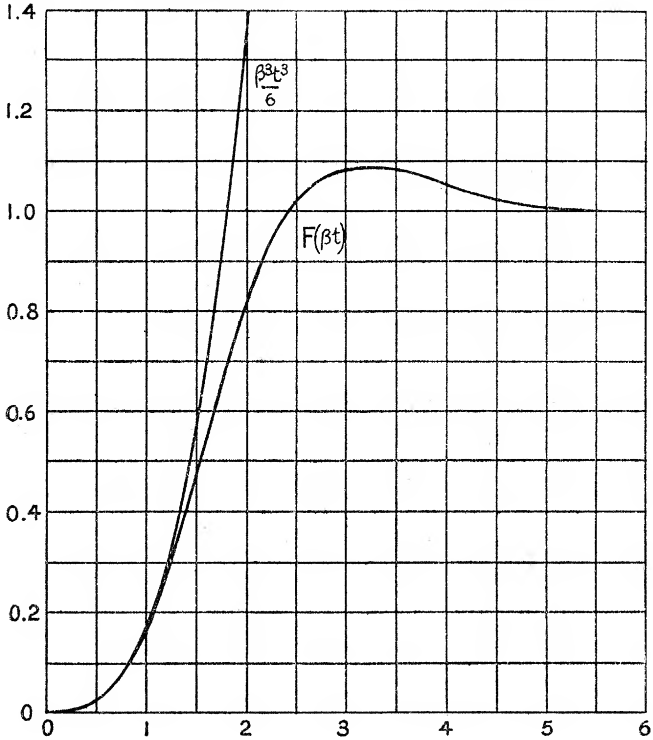

FIG. 5

If, instead of a continuous plate, there are a number of equally spaced rectangular rods, as shown in fig. 6 (*a*), the path of the flux is successively through metal and air, but if the air spaces are narrow, the paths of the lines of force will not deviate appreciably from straight parallel lines, as shown in fig. 6 (*b*). With round wires, which are, of course, always employed, and with a low space-factor, the flux distribution will be somewhat as shown in fig. 6 (*c*). It is assumed, however, for the purposes of calculation, that the round wires are replaced by square ones of equal cross-section, and that the flux is as shown in fig. 6 (*b*). On this assumption the only effect of the air spaces will be to cause a deeper penetration of the flux into the metal. If

$$\frac{\text{path of flux in metal}}{\text{total length of path}} = \psi,$$

then, $H_1 = 4\pi i_s \psi/10$, where i_s is the fictitious line current per centimetre width of conductor. This is equivalent to decreasing the permeability from

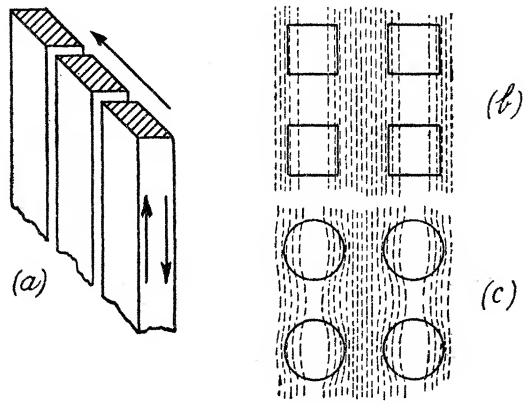

FIG 6

1 to ψ, in that it decreases the value of L in this ratio if the permeability of the metal is unity; if it is μ, the equivalent reduced value is $\mu\psi/(\psi+\mu-\mu\psi)$. With this modification the formulæ for the continuous sheet can be employed and the air spaces neglected.

Assuming in what follows that the metal is non-magnetic, β will be replaced by $\beta\sqrt{\psi}$, and the power dissipated in the metal will be

$$H_1^2\left(\frac{10}{4\pi}\right)^2 \frac{\beta\rho\sqrt{\psi}}{t\psi^2} F(\beta t\sqrt{\psi}) \text{ watts per cubic centimetre.} \qquad (17)$$

If the conductors are square in cross-section and uniformly spaced, as shown in figs. 7 (*a*) or 7 (*b*), the space-factor $\alpha = \psi^2$, and if the cross-section of each wire is t^2, the power dissipated per centimetre length of each wire

$$= H_1^2\left(\frac{10}{4\pi}\right)^2 \cdot \frac{\beta\rho t}{\psi^{3/2}} \cdot F(\beta t\sqrt{\psi}),$$

$$= H_1^2\left(\frac{10}{4\pi}\right)^2 \frac{\beta\rho t}{\alpha} \alpha^{1/4} F(\beta t\alpha^{1/4}) \text{ watts per centimetre of wire.} \qquad (18)$$

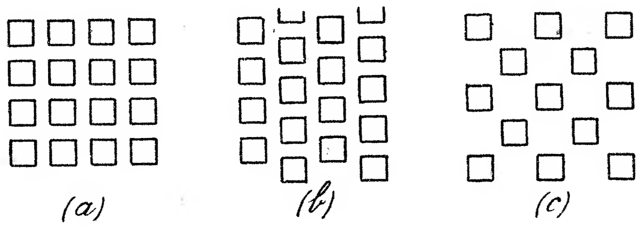

FIG 7

If the wires are spaced as shown in fig. 7 (*c*), $\alpha = 2\psi^2$, and the power dissipated

$$= 1{\cdot}68\,\mathrm{H}_1^2\left(\frac{10}{4\pi}\right)^2\frac{\beta\rho t}{\alpha}\,.\,\alpha^{1/4}\,\mathrm{F}\left(\frac{\beta t\alpha^{1/4}}{1{\cdot}19}\right) \text{ watts per centimetre of wire.} \tag{18a}$$

To find the average value of the loss due to eddy-currents in a straight stranded conductor of circular cross-section carrying an alternating current with a maximum value of I amperes, it is necessary to substitute for H_1^2 its average value throughout the whole cross-section. This has already been shown to be

$$4\mathrm{I}^2/50\,\mathrm{D}^2 \qquad \text{(see equation 2).}$$

The average loss due to eddy-currents per centimetre of individual wire, assuming equation 18

$$= \frac{4}{50}\cdot\left(\frac{10}{4\pi}\right)^2\cdot\frac{\mathrm{I}^2}{\mathrm{D}^2}\cdot\frac{\beta\rho t}{\alpha}\,.\,\alpha^{1/4}\,\mathrm{F}\,(\beta t\alpha^{1/4}) \text{ watts.}$$

The loss due to the main current I/n per centimetre of individual wire

$$= \frac{\mathrm{I}^2}{2n^2}\cdot\frac{\rho}{t^2} \text{ watts,}$$

and therefore the total loss per centimetre of individual wire

$$= \frac{\mathrm{I}^2}{2n^2}\left[\frac{\rho}{t^2}+\frac{n^2}{\pi^2\mathrm{D}^2}\cdot\frac{\beta\rho t}{\alpha}\,.\,\alpha^{1/4}\,\mathrm{F}\,(\beta t\alpha^{1/4})\right].$$

Hence
$$\mathrm{R}_f/\mathrm{R}_o = 1+\frac{n\beta t}{4\pi}\,.\,\alpha^{1/4}\,\mathrm{F}\,(\beta t\alpha^{1/4}),$$

or, substituting for β,

$$\mathrm{R}_f/\mathrm{R}_o = 1+\frac{nt}{2}\sqrt{\left(\frac{f}{10^9\rho}\right)}\,.\,\alpha^{1/4}\,\mathrm{F}\,(\beta t\alpha^{1/4}). \tag{19}$$

For values of $\beta t\alpha^{1/4}$ less than unity, the approximate value, viz. $\mathrm{F}\,(x) = x^3/6$, can be substituted; in this case

$$\mathrm{R}_f/\mathrm{R}_o = 1+\frac{nt}{2}\sqrt{\left(\frac{f}{10^9\rho}\right)}\,\alpha^{1/4}\beta^3t^3\alpha^{3/4},$$

$$= 1+\frac{n\alpha t^4\beta^4}{24\pi}.$$

For copper $\rho = 1{\cdot}7\,.\,10^{-6}$ and

$$\mathrm{R}_f/\mathrm{R}_o = 1+7{\cdot}15\,.\,10^{-6}nf^2t^4\alpha.$$

Substituting round wires of diameter d of equal cross-sectional area for the square wires of side t, so that

$$\pi d^2/4 = t^2,$$

$$\mathrm{R}_f/\mathrm{R}_o = 1+4{\cdot}4\,.\,10^{-6}nf^2d^4\alpha, \tag{20}$$

which agrees with the value calculated in Part I, equation ($3a$), except that the constant was there found to be 4·2.

In a single-layer solenoid, wound with S turns per centimetre of a stranded conductor of square cross-section with side τ, the average value of H_1^2 over the cross-section has been shown to be

$$\frac{16\pi^2 I^2 S^2}{300} \quad \text{(see equation 4).}$$

Substituting this value of H_1^2, the loss due to eddy-currents in each individual wire

$$= \frac{I^2 S^2 \beta \rho t}{3\alpha} \alpha^{1/4} F(\beta t \alpha^{1/4}) \text{ watts per centimetre,}$$

whilst the loss due to the main current of I/n is

$$\frac{I^2}{2n^2} \frac{\rho}{t^2}.$$

Hence, putting $nt^2 = \alpha\tau^2$, we have

$$R_c/R_o = 1 + \tfrac{2}{3}(S\tau)^2 n\beta t\alpha^{1/4} F(\beta t\alpha^{1/4}),$$

or, substituting for β,

$$R_c/R_o = 1 + \frac{4\pi}{3}(S\tau)^2 nt \sqrt{\left(\frac{f}{10^9\rho}\right)} \alpha^{1/4} F(\beta t\alpha^{1/4}). \tag{21}$$

For values of $\beta t\alpha^{1/4}$ less than unity this becomes

$$R_c/R_o = 1 + \tfrac{1}{9}(S\tau)^2 n\alpha t^4\beta^4.$$

For copper this reduces to

$$R_c/R_o = 1 + 59{\cdot}4 \,.\, 10^{-6} \,.\, nf^2t^4\alpha (S\tau)^2,$$

or, substituting round wires of equal cross-section,

$$R_c/R_o = 1 + 36{\cdot}7 \,.\, 10^{-6} \,.\, nf^2d^4\alpha (S\tau)^2, \tag{22}$$

which agrees with the value found in Part I, equation $5b$, except that the constant was there found to be 35·6.

Equations 20 and 22 are independent of any assumption as to the arrangement of the wires, and can be obtained either from equation 18 or $18a$.

Hence the method adopted in Part I is permissible, and the values given in Tables I–VI are correct for values of $\beta t\alpha^{1/4}$ less than unity. For copper, $\beta = 0{\cdot}152\sqrt{f}$, and, assuming $\alpha = 0{\cdot}5$, t must not exceed $7{\cdot}9\sqrt{f}$. If round wires of diameter d are substituted for square wires of side t, their cross-sections being equal, the largest sizes of wires at various frequencies, for which the method of Part I is applicable, are as follows :—

$f =$	10^6,	$5 \,.\, 10^5$,	$2{\cdot}5 \,.\, 10^5$,	10^5,	$5 \,.\, 10^4$,
$d =$	0·009,	0·013,	0·018,	0·028,	0·04 cm.

For smaller values of α than 0·5, the limiting values of d can be further increased. Hence Tables I, II, IV, and V can be relied upon at all frequencies, and Tables III and VI at frequencies below $2 \cdot 10^5$, *i.e.* for wave-lengths exceeding 1500 metres. It is only with the largest wire, viz., $d = 0{\cdot}02$ cm., at the higher frequencies, that the screening action of the eddy-currents causes an appreciable diminution in the eddy-current losses, and gives values of R_f/R_o or R_c/R_o smaller than those given in the Tables. A comparison of the columns headed B and C for $f = 10^6$ shows, however, that, even if the high-frequency resistance of the stranded conductor were reduced to half the value there given, it would still offer no advantage over the solid wire which could replace it. An accurate calculation of the first example in Table III gives $R_f/R_o = 1{\cdot}66$ instead of 2. If the accurate formula involving $F(\beta t \alpha^{1/4})$ is used to determine the best value of the space factor, the results obtained differ very little from those given in the Tables.

In establishing the various formulæ in this paper, it has been tacitly assumed that each individual wire is situated in a uniform magnetic field, except for the effects of the eddy-currents in the wire. This is not strictly true, since the field increases in intensity with increasing distance from the centre in the case of a straight conductor, and with increasing distance from the outer surface in the case of a solenoid. This introduces a dissymmetry in the distribution of the eddy-currents within each wire, and thereby increases the losses and the high-frequency resistance. Any attempt to take this secondary effect into account would greatly complicate the equations, and is thought unnecessary.